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ON INERTIAL EFFECTS ON DISCONTINUITIES IN THE CONCENTRATION OF THE SOLID PHASE IN A DISPERSE MEDIUM*

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A system of conditions for the conservation of mass and momentum of a fluid and a solid phase on the surface of discontinuity in a disperse medium is analysed within the framework of the double continuum model /1/. The case when there is a high concentration of solid particles on one side of the discontinuity and a low or zero concentration of them on the other side is considered. Under these conditions in the region of a high concentration of the solid phase remote from the discontinuity, the inertial force of the fluid phase is small compared with the interphase interaction force and Darcy's law holds while both forces are of the same order of magnitude in the thin transition lower close to the surface of discontinuity. It is assumed that the surface of discontinuity is impermeable to the particles of the solid phase. Effects due to the possible occurrence of surface tension on the discontinuity are not considered.

Subject to the assumptions which have been made, a solution of the initial system of equations of motion and continuity of the phases is constructed in the stationary approximation taking account of the transition layer which satisfies the condition of the continuity of the

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pressure and the velocity of the fluidizing agent on the boundaries of the layer. Replacement of the transition layer by a discontinuity leads to a jumpwise change in the above-mentioned parameters upon crossing the discontinuity which, in the case being considered, necessitates a modification of the whole system of boundary conditions on the surface of discontinuity /1-3/. Examples of the construction of the flow fields of the fluid and solid phases during the motion of a local inhomogeneity in the concentration of the particles in the pseudofluidized layer are presented.

Inertial (dynamic) effects on the discontinuities during the slow filtration of a gas in porous media have been considered previously /4, 5/. The inertialess model of a fluidizing agent has been used (6, 7/, etc.) to analyse the motion of bubbles in a pseudofluidized bed.

1. Formulation of the problem. In the continuum approximation a fluidizing agent and a collection of solid particles are phenomenologically modelled by two mutually permeable interacting continuous media (a fluid phase and a solid phase respectively). For simplicity, let us assume that they possess the properties of a ideal fluid and that the viscosity of the fluidizing agent is only manifested on a micro level and, after the averaging procedure, only occurs in the initial equations in a term which describes the interaction between the phases. We assume that the densities d_f and d_s of the fluidizing agent and the particles are constant. Then, the locally averaged equations for the conservation of momentum and mass of the fluid and solid phases can be written in the form /8/

$$
\begin{aligned}
&d_f\varepsilon\left[\frac{\partial}{\partial t}+(\mathbf{v}\nabla)\right]\mathbf{v}=-\nabla p_f+d_f\varepsilon\mathbf{g}-\mathbf{f},\quad \frac{\partial\varepsilon}{\partial t}+\nabla(\varepsilon\mathbf{v})=0\\
&d_s\rho\left[\frac{\partial}{\partial t}+(\mathbf{w}\nabla)\right]\mathbf{w}=-\nabla p_s+d_s\rho\mathbf{g}+\mathbf{f},\quad \frac{\partial\rho}{\partial t}+\nabla(\rho\mathbf{w})=0\\
&\varepsilon+\rho=1,\quad \Phi(p_s,\rho,\ldots)=0
\end{aligned}
\tag{1.1}
$$

Here $\mathbf{v}$ and $\mathbf{w}$ are velocities, p_f and p_s are the pressures of the fluid and solid phases, ρ is the concentration of the solid phase (ε is the porosity of the system) and $\mathbf{g}$ is the acceleration due to an external mass force (gravitational force). The last of the equation in (1.1) is the "equation of state" of the solid phase.

We take the two-term expression

$$
\mathbf{f}=-\rho\nabla p_f-\rho(\mathbf{w}-\mathbf{v})F(|\mathbf{w}-\mathbf{v}|,\rho)
\tag{1.2}
$$

for the force due to the interaction between the phase $\mathbf{f}$. In particular, this expression holds in the case of a suspension of particles in a gas when $d_f/d_s \ll 1$. In doing this, the effects of the connected masses are not taken into account while the second term in (1.2) is the inperphase frictional force without any allowance for the Besset force.

Let us assume that the distribution of the concentration of the solid phase $\rho=\rho(\mathbf{r},t)$ changes in a jumpwise manner on crossing a certain surface which approximates a thin transition layer Δ in which the parameters of the system undergo a sharp change.

Let us now introduce a rectangular coordinate system xyz associated with an infinitely small planar element of the surface of discontinuity where z is measured along the normal to the surface, and agree to mark with a dash values which refer to the flow region $z<0$. We shall denote the jump in any parameter on passing from the region $z>0$ to the region $z<0$ by square brackets.

Let us write the system of boundary conditions on the discontinuity which has been obtained previously /1-3/ from the integral equations for the mass and momentum balances for the fluid and solid phases in the surface of discontinuity encompassed by an elementary volume on passing to the limit as $\Delta\to 0$ in the form

$$
\begin{aligned}
&z=0,\quad [\varepsilon v_z]=0,\quad [\rho w_z]=0\\
&\varepsilon v_z[\mathbf{v}_\tau]=0,\quad \rho w_z[\mathbf{w}_\tau]=0,\quad [p_f+{}^1\!/_2 d_f v_z^2]=0\\
&[p_f+p_s+d_f\varepsilon v_z^2+d_s\rho w_z^2]=0\quad ([\zeta]=\zeta'-\zeta)
\end{aligned}
\tag{1.3}
$$

($\mathbf{v}_\tau$ and $\mathbf{w}_\tau$ are the components of the velocities of the phases which are tangential to the surface of discontinuity).

The system of analogous conditions in the inertialess fluid phase model ($d_f=0$) has the form

$$
\begin{aligned}
&z=0,\ [\varepsilon v_z]=0,\ [\rho w_z]=0,\ \rho w_z[\mathbf{w}_\tau]=0\\
&[p_f]=0,\quad [p_f+p_s+d_s\rho w_z^2]=0
\end{aligned}
\tag{1.4}
$$

Conditions (1.3) and (1.4) hold for a planar discontinuity and also for an element of the surface of discontinuity of arbitrary curvature when there is no surface tension.

The fact that, in system (1.4), there is no boundary condition on the tangential component of the velocity of the fluid phase is associated with the elimination of the convective term with the highest (in the given case, first) derivative of the velocity in the inertialess approximation. On account of the smallness of the inertial term outside the transition layer of thickness Δ which is adjacent to the discontinuity, the third condition (1.3) (obtained when $\Delta \to 0$) may be considered as the condition for the continuity of the tangential component of the velocity of the fluid on the boundaries of the transition layer.

The system of conditions (1.3) and (1.4) must be augmented by considerations concerning the nature of the discontinuity.

Let us confine ourselves to the treatment of discontinuities of fixed form in the disperse system with a concentration distribution of the solid phase which is constant in time (in the coordinate system associated with the discontinuity). At large distances from the discontinuity $z = 0$, the system is homogeneous with a concentration of particles ρ when $z > 0$ and ρ' when $z < 0$. A similar condition was assumed to hold /6/ in the analysis of the motion of a single gas bubble in a pseudofluidized layer and corresponds to the simplest "equation of state" of a solid phase in the form $\rho = \mathrm{const}$.

We will approximate the interaction force between the phases by an expression which is linear with respect to the velocity of relative motion of the particles and the fluid. In this case the function $F(|\mathbf{w} - \mathbf{v}|, \rho)$ in formula (1.2), subject to all the particles being identical and of spherical form, can be represented in the form /8/

$$F(|\mathbf{w} - \mathbf{v}|, \rho) \equiv F(\rho) = K(\rho)\tau_0^{-1} d_s \tag{1.5}$$
$$K(\rho) = (1-\rho)^{-11/4}, \quad \tau_0 = {}^2/_9 a_p^2 \gamma / \nu_f, \quad \gamma = d_s/d_f \gg 1$$

(τ_0 is the relaxation time of the velocity of particles of radius a_p in a current of gas with a coefficient of kinematic viscosity ν_f). The motion of the phases is assumed to be a stationary in the coordinate system associated with the discontinuity.

Let us now write down the stationary equations of motion and the equations of continuity of the phases in dimensionless form (retaining the previous notation for the dimensionless quantities). In order to do this, we introduce the following parametric scales for the biphasic flow: v_0 and U_s are the velocities of the fluid and solid phases, l are lengths, J and J' are the pressure gradients of the fluid phase in the regions $z > 0$ and $z < 0$ and P and P' are the pressures of the disperse phase in the regions $z > 0$ and $z < 0$.

For example, in the case of a pseudofluidized layer, v_0 is the velocity of pseudofluidization in the region of the homogeneous layer remote from the discontinuity /8/: $v_0 = v_0(\rho) = (1-\rho) d_s g [F(\rho)]^{-1}$, V_s is the velocity of propagation of the discontinuity in the layer which, in the case of the motion of a bubble, for example, is equal to its velocity of ascent in the layer, J is the weight of a unit volume of the layer in the region $z > 0$: $J = (d_f\varepsilon + d_s\rho) g$ (and, respectively, $J' = (d_f\varepsilon' + d_s\rho') g$ when $z < 0$). In the problem of the motion of a gaseous piston in an apparatus with a fluidized bed, the diameter of the apparatus or the local radius of curvature of the surface of discontinuity may be taken as the linear scale l, while in the problem of the motion of a bubble or packet in a boundaryless psuedofluidized bed one may take the corresponding characteristic size of the region of inhomogeneity.

In the stationary approximation and dimensionless variables, Eqs.(1.1) take the form

$$\begin{aligned}
&z > 0, \quad \mathrm{Ar}^* (\mathbf{v}\nabla)\mathbf{v} = -(\varepsilon(\gamma\rho)^{-1} + 1)\nabla p_f + (\gamma\rho)^{-1}\mathbf{i}_g + \delta\mathbf{w} - \mathbf{v} \\
&\mathrm{Fr}^* (\mathbf{w}\nabla)\mathbf{w} = -P_s\nabla p_s - (\rho\varepsilon^{-1} + \gamma^{-1})\nabla p_f + \varepsilon^{-1}\mathbf{i}_g - \delta\mathbf{w} + \mathbf{v} \\
&\nabla\mathbf{v} = 0, \quad \nabla\mathbf{w} = 0, \quad \rho = \mathrm{const} \\
&z < 0, \quad \mathrm{Ar}^{*\prime} (\mathbf{v}'\nabla)\mathbf{v}' = -\omega_f(\varepsilon'(\gamma\rho')^{-1} + 1)\nabla p_f' + \omega_f(\gamma\rho')^{-1}\mathbf{i}_g + \\
&\quad \delta\mathbf{w}' - \mathbf{v}' \\
&\mathrm{Fr}^{*\prime} (\mathbf{w}'\nabla)\mathbf{w}' = -P_s'\nabla p_s' - \omega_s\varepsilon^{-1}(\rho' + \varepsilon'\gamma^{-1})\nabla p_f' + \\
&\quad \omega_s\varepsilon^{-1}\mathbf{i}_g - \delta\mathbf{w}' + \mathbf{v}' \\
&\nabla\mathbf{v}' = 0, \quad \nabla\mathbf{w}' = 0, \quad \rho' = \mathrm{const}, \quad \mathbf{i}_g = \mathbf{g}/g
\end{aligned} \tag{1.6}$$

The dimensionless parameters are related to the parameters of the disperse system in the following manner:

$$\mathrm{Ar}^* = \frac{(1-\rho)^{11/2}}{162\rho}\mathrm{Ar}\frac{a_p}{l}, \quad \mathrm{Ar}^{*\prime} = \sigma^{-1}\mathrm{Ar}^* \tag{1.7}$$
$$\mathrm{Ar} = \frac{8a_p^3 g\gamma}{\nu_f^2}, \quad \omega_f = \left(\frac{1-\rho'}{1-\rho}\right)^{15/4}, \quad \sigma = \frac{\rho'}{\rho\omega_f}$$

$$\mathrm{Fr}^* = \frac{U_s^2}{gl(1-\rho)}, \quad \mathrm{Fr}^{*\prime} = \omega_s\mathrm{Fr}^*, \quad \omega_s = \left(\frac{1-\rho'}{1-\rho}\right)^{11/4} \tag{1.8}$$

$$P_s = \frac{P}{d_s\rho g l\varepsilon}, \quad P_s' = \frac{P'\omega_s}{d_s\rho' g l\varepsilon}, \quad \delta = \frac{U_s}{v_0} \tag{1.9}$$

Here, Ar^*, $Ar^{*\prime}$, Fr^* and $Fr^{*\prime}$ are modified Archimedes' and Froude numbers in the regions $z > 0$ and $z < 0$, Ar is the Archimedes' number.

2. Analysis of the boundary conditions. Model of the disperse phase. We shall analyse the general conditions of conservation (1.3) on the concentration discontinuity in the pseudofluidized layer on the basis of the dimensionless equations of motion (1.6) and relationships (1.7)-(1.9). In accordance with the statements in Sect.1, we shall confine ourselves to the treatment of the case when the bulk concentration of the solid particles on one side of the discontinuity is small and that on the other side is large. At the same time we assume a concentration of particles in the region of the rarified layer which is so small that the inertial force of the fluid phase and the interaction force between the two phases in this region are of the same order of magnitude. To be specific, let $\rho' \ll 1$ in the region $z < 0$ and let the layer be dense in the region $z > 0$ so that $\rho \cong 0.5$. The modified Archimedean criteria Ar^* and $Ar^{*\prime}$ which occur in the dimensionless equations for the conservation of momentum of the fluid phase characteristize the relative contribution of the inertial terms compared with the forces arising from the interaction between the phases. It follows from physical considerations that, as the concentration of the disperse phase decreases in one of the regions of flow, the inertia of the fluid phase becomes substantial and influences the formation of the structure of the flows of the phases in the region under consideration. We note that, also, the contribution of the inertial term cannot be neglected in the limiting case when there are no solid particles along one of the sides of the discontinuity such as, for example, in the analysis of the flow field within bubbles in a pseudofluidized bed.

Let us now estimate the dimensionless parameters in Eqs.(1.6) on the basis of the following model representations regarding the disperse particles of the pseudofluidized layer. We shall assume that the particles are fairly fine: $a_p/l = \lambda \ll 1$ (which corresponds to the continuum approach), that they are heavy: $\gamma \gg 1$, and that the velocity of pseudofluidization of particles with such properties is finite. We will write down the last condition, taking account of the relationship for the velocity of pseudofluidization (Sect.1) and allowing for the fact that $U_s \sim (gl)^{1/2}$ /8-10/ in the form

$$\delta = \frac{U_s}{v_0} \sim (gl)^{1/2} \frac{\nu_f}{a_p^2 \gamma g} \sim \frac{1}{G_*^{1/2} \lambda^2 \gamma} \sim 1, \quad G_* = \frac{g l^3}{\nu_f^2} \tag{2.1}$$

Together with the assumptions that $\lambda \ll 1$, $\gamma \gg 1$, we assume that $G_* \gg 1$ ($G_* = 10^5 - 10^8$ in real systems). Together with the small parameters λ, γ^{-1} and G_*^{-1}, we shall consider the concentration of particles ρ' in the region of the rarified layer $z < 0$ as the fourth independent small parameter of the problem. The assumption which has been made in this model concerning the linearity of the interaction between the phases (which corresponds to small Reynolds numbers Re_p for an individual solid particle) remains approximately valid up to extremely high values of the Archimedes' number: the constraint $Ar \lesssim 20\rho^{-19/4} \cong 10^2 - 10^3$, $\rho \gtrsim 0.3$ (pseudofluidization in the dense layer) corresponds to the condition $Re_p = 2a_p v_0(\rho)/\nu_f \lesssim 1$ /8, 11/. On account of this, we put $Ar \sim \lambda^{-m} \gg 1$, where $0 < m < 1$ (in the case when $m = 0$ $Ar \sim 1$). On the basis of relationships (1.7) and (2.1), we have $Ar = 8G_*\lambda^3\gamma \sim G_*^{1/2}\lambda \sim \lambda^{-m}$. This condition provides a link between the parameters G_* and λ in the form $G_* \sim \lambda^{-2(m+1)}$.

In the region with a low concentration of the solid phase, where the inertial force and the force due to interaction between the phases are, according to the assumption, quantities of the same order of magnitude, the modified Archimedes' number $Ar^{*\prime} \sim 1$. Using the second equality of (1.7) we obtain an expression for the parameter $Ar^{*\prime}$ in the region $z < 0$ when $\rho' \ll= 1$ in the form $Ar^{*\prime} \sim G_*\gamma\lambda^4/\rho' \sim 1$. The relationship between the parameters ρ' and λ: $\rho' \sim \lambda^{1-m}$ follows from the latter expression.

Next, we shall have

$$\sigma \sim \lambda^{1-m} \ll 1, \quad \omega_{f,s} \sim 1, \quad Ar^* \sim \lambda^{1-m} \ll 1$$
$$\gamma\rho \sim \lambda^{m-1} \gg 1, \quad \gamma\rho' \sim 1$$

At the same time, we obtain

$$\varkappa \sim \rho^{-1/3}\lambda \sim \lambda \ll 1, \quad \varkappa' \sim \rho'^{-1/3}\lambda \sim \lambda^{(m+2)/3} \ll 1$$

for the dimensionless interparticle distances $\varkappa$ and $\varkappa'$ in the regions $z > 0$ and $z < 0$ respectively.

Hence, in the case of the type of discontinuities being considered, the problem of determining the flow field of the fluid phase in the region of the dense layer is characterized by the fact that the initial equation of motion of the gas contains the small parameter $Ar^* \sim \lambda^{1-m}$ accompanying the highest derivative of the velocity. Hence, in the dense layer remote from the discontinuity (on length scales comparable with l), the inertia of the gas is negligibly small compared with the friction between the phases so that the order of the equation, describing the filtration of a gas in a fluidized bed with an accuracy up to the highest terms in the small parameter Ar^*, is reduced. Actually, at distances which are remote from the discontinuity, we obtain $\nabla p_f + \delta\mathbf{w} - \mathbf{v} = 0$ from the first equation of (1.6), which corresponds to Darcy's law.

Meanwhile, together with the simplification of the initial equation in the main bulk of the dense part of the layer, the need arises for a treatment close to the surface of discontinuity of the transition layer (the analogue of a Prandtl boundary layer) in which the inertial term is substantial and occurs together with the term describing the interaction between the phases. The thickness Δ of the transition zone is of the order of the modified Archimedes' number in the region of the dense layer: $\Delta \sim \mathrm{Ar}^*$. Similar inertial effects in the vicinity of boundary surfaces during slow filtration in porous media have been investigated previously in /4/.

Using the second equality of (1.7) we shall write the equation of motion of the fluid phase in the region $z < 0$ in the form

$$\mathrm{Ar}^* (\mathbf{v}'\nabla)\,\mathbf{v}' = -\,\omega_f \sigma\,(\varepsilon'/(\gamma\rho')^{-1} + 1)\,\nabla p_f' + (\gamma\rho)^{-1}\,\mathbf{i}_g + \sigma(\delta\mathbf{w}' - \mathbf{v}') \tag{2.2}$$

All the terms in (2.2) are of the same order of magnitude: $O(\mathrm{Ar}^*)$. Consequently, making allowance for the inertia of the gas in the region where there is a low concentration of disperse particles also implies that the term $O(\mathrm{Ar}^*)$ should be retained in the equation of motion of the fluid phase in the dense layer and that a boundary condition should be imposed on the jump in pressure of the gas on the discontinuity in the form $[p_f + {}^1/_2\, d_f v_z^2] = 0$. In the zeroth approximation with respect to the small parameter $\mathrm{Ar}^* \sim \lambda^{1-m}$, the pressure jumps are constrained by an analysis of Darcy's equation in the dense part of the layer by putting $p_f = 0,\ [p_f] = 0$ on the surface of discontinuity /6/.

Hence, the construction of the flow fields and the pressure distributions of the phases in a disperse system with a "strong" concentration discontinuity infers the integration of the initial Eqs.(1.6) both within the transition layer and outside of it with a subsequent matching of the solutions on the provisional boundaries of the transition region. Within the framework of a model which permits a continuous change in the concentration of the disperse phase in the transition layer from ρ' to ρ, the solutions obtained on matching must satisfy the requirement of the continuity of the parameters of a biphasal flow. When the thickness of the transition layer is neglected, these parameters will change abruptly on crossing the surface of discontinuity.

The modified Froude number $\mathrm{Fr}^* \sim 1$ in the equations of motion of the solid phase in (1.6). Actually, numerous experiments and theoretical estimates show that during the motion of bubbles and gaseous plugs, for example, their velocity of ascent in the pseudofluidized layer is $U_s \sim (gl)^{1/2}$. It is important to emphasize that, according to Eqs.(1.8), the ratio of the magnitude of the inertial term in the solid phase to the force due to the interaction between the phases is independent of the concentration of particles in the layer, i.e. $\mathrm{Fr}^*/\mathrm{Fr}^{*\prime} \sim 1$ always. We shall also assume that the similarity numbers P_s and P_s' are of the order or unity.

3. Motion of the phases in the transition layer.

As in boundary layer theory, we shall assume that the velocity fields and the pressure distributions of the phases outside the transition region are known from the solution of the corresponding problem on the motion of two-phase flow with a concentration discontinuity.

Let us now introduce the new dimensionless coordinates η, ϑ and n in the transition layer region using the formulae

$$\eta = x,\ \ \vartheta = y,\ \ n = \mathrm{Ar}^{*-1} z\ (\eta,\ \vartheta,\ n \sim 1) \tag{3.1}$$

where η and ϑ are tangential coordinates and n is an elongated normal to the boundary of the coordinate. In accordance with (1.6), it is advisable to choose the magnitude of the pressure difference on the discontinuity $[p_f] \sim d_f v_0^2$ as the scale of pressure of the fluid phase in the transition layer (the other scales remain the same as in the derivation of Eqs.(1.6)).

By writing the dimensionless initial Eqs.(1.1) $(\partial/\partial t = 0)$ in the variables (3.1) and comparing the orders of the terms, we arrive at the following system of equations of motion and phase continuity in the transition layer:

$$\begin{gathered}
v_n^\circ \frac{\partial \mathbf{v}_\tau^\circ}{\partial n} = \sigma(\rho^\circ, \rho)(\delta \mathbf{w}_\tau^\circ - \mathbf{v}_\tau^\circ) \\
v_n^\circ \frac{\partial v_n^\circ}{\partial n} = -\frac{\partial p_f^\circ}{\partial n} + \sigma(\rho^\circ, \rho)(\delta w_n^\circ - v_n^\circ) \\
w_n^\circ \frac{\partial \mathbf{w}_\tau^\circ}{\partial n} = 0, \quad \frac{\partial}{\partial n}\left(\mathrm{Fr}^* \frac{w_n^{\circ 2}}{2} + P_s^\circ p_s^\circ\right) = 0 \\
\frac{\partial}{\partial n}(\varepsilon^\circ v_n^\circ) = 0, \quad \frac{\partial}{\partial n}(\rho^\circ w_n^\circ) = 0 \\
\rho^\circ = \rho^\circ(\rho, \rho') = \begin{cases} \rho, & n \to \infty \\ \rho', & n \to 0 \end{cases}, \quad \sigma(\rho^\circ, \rho) = \begin{cases} 1, & n \to \infty \\ O(\mathrm{Ar}^*), & n \to 0 \end{cases}
\end{gathered} \tag{3.2}$$

Here $\mathbf{v}_\tau^\circ$ and $\mathbf{w}_\tau^\circ$ are the components of the velocities of the phases which are tangential to the discontinuity (see (1.3)), $\rho^\circ(\rho, \rho')$ is a certain function of its arguments, possessing the above-mentioned properties (essentially, it is the equation of state of the solid phase in the transition layer). The flow parameters in the region of the transition layer in (3.2) and, subsequently, everywhere are labelled with the superscript $^\circ$.

We write the conditions for the asymptotic matching of the solution of system (3.2) to the solution outside the transition layer in the form

$$\begin{aligned} n \to \infty, \quad & A^\circ(\eta, \vartheta, n) \to A(\eta, \vartheta, 0) = \lim_{z \to +0} A(x, y, z) \\ n \to 0, \quad & A^\circ(\eta, \vartheta, n) \to A'(\eta, \vartheta, 0) = \lim_{z \to -0} A(x, y, z) \end{aligned} \tag{3.3}$$

where the symbol A denotes any of the three quantities: v_n, $\mathbf{v}_\tau$, p_f. In the last case $|A(\eta, \vartheta, 0)| = \infty$ by virtue of the fact that we have $d_s \rho g l/(d_f v_0^2) = O(\mathrm{Ar}^{*-1}) \gg 1$ for the ratio of the magnitudes of the scales of the fluid phase pressure in the dense layer and the transition region.

Let us now consider a number of special cases in which system (3.2) is simplified.

1°. *Filtration in a homogeneous porous body* ($\mathbf{w} \equiv 0$, $\rho^\circ = \mathrm{const} = \rho$, $\sigma(\rho^\circ, \rho) = 1$). The transition layer Eqs.(3.2) take the form /4/

$$v_n^\circ \frac{\partial \mathbf{v}_\tau^\circ}{\partial n} = -\mathbf{v}_\tau^\circ, \quad \frac{\partial p_f^\circ}{\partial n} = -v_n^\circ, \quad \frac{\partial v_n^\circ}{\partial n} = 0 \tag{3.4}$$

Integration of system (3.4) yields

$$\begin{aligned} & v_n^\circ = l(\eta, \vartheta), \quad p_f^\circ = -f(\eta, \vartheta)\, n + \varphi(\eta, \vartheta) \\ & \mathbf{v}_\tau^\circ = \Psi(\eta, \vartheta) \exp(-n/f(\eta, \vartheta)) \end{aligned} \tag{3.5}$$

The unknown functions $f(\eta, \vartheta)$, $\varphi(\eta, \vartheta)$ and $\Psi(\eta, \vartheta)$ must be determined from the matching conditions (3.3). It is obvious that $f(\eta, \vartheta) = v_n(\eta, \vartheta, 0)$. We note that, on segments of the transition layer surface when $f(\eta, \vartheta) > 0$, the fluid floes into the region of the dense layer. On the other hand, when $f(\eta, \vartheta) < 0$, on a part of the surface of discontinuity the fluid flows out of the region of the dense layer into the rarified region.

In the model of the discontinuity being discussed it is not possible to satisfy condition (3.3) for the normal component of the velocity of the fluid phase as $n \to 0$. In this case, there will be a jump in the normal component of the velocity equal to

$$v_n'(\eta, \vartheta, 0) - v_n(\eta, \vartheta, n) = f(\eta, \vartheta)(\varepsilon/\varepsilon' - 1)$$

on the boundary of the transition layer with the region containing a low concentration of particles.

The corresponding jump in the pressure of the fluidizing agent on the boundary $n = 0$ is equal to $[p_f] = -[{}^1/_2\, v_n^2]$. The latter condition determines the function $\varphi(\eta, \vartheta)$ in the form

$$\varphi(\eta, \vartheta) = {}^1/_2 f^2(\eta, \vartheta)(\varepsilon^2/\varepsilon'^2 - 1) + p_f'(\eta, \vartheta, 0)$$

It follows from the second condition of (3.5) that, if $f(\eta, \vartheta) > 0$ $(f < 0)$, the gradient of the pressure in the transition layer is negative (positive) which corresponds to a fall (increase) in the pressure when the gas flows into the dense layer (when the gas flows out of the region of the dense layer).

It follows from the third equation of (3.5) and the matching conditions (3.3) that, by virtue of the strong constraints imposed on system (3.2) within the framework of the assumed model, the distribution of the tangential velocity of the gas on segments where it flows in cannot be continuously matched to both the distributions $\mathbf{v}_\tau(\eta, \vartheta, 0)$ and $\mathbf{v}_\tau'(\eta, \vartheta, 0)$ outside of the transition region. Since, when this is so, $\mathbf{v}_\tau^\circ(\eta, \vartheta, n) \to \infty$ $(n \to \infty)$, $f(\eta, \vartheta) < 0$ which is physically unreal, a transition region is not formed on segments where there is a flow out of the fluidizing agent from the dense of the layer (cf. /4/).

2°. *Discontinuity in the concentration of the disperse phase in a pseudofluidized layer.* Let us now consider a disperse system of more complex form, a pseudofluidized layer ($\mathbf{w} \not\equiv 0$). Let us suppose that the surface of discontinuity is impermeable to the solid particles, i.e. $w_n = w_n' = 0$, $z = 0$. For example, the boundary between a fluidized bed and a region filled with the pure fluidizing agent constitutes a discontinuity of this type. As in the previous model, let $\rho^\circ = \mathrm{const} = \rho$ $(\sigma(\rho^\circ, \rho) = 1)$. When $\rho^\circ = \mathrm{const}$, it follows from the penultimate equation of system (3.2) that $\partial w_n^\circ/\partial n = 0$, whence $w_n^\circ(\eta, \vartheta) = w_n(\eta, \vartheta, 0) \equiv \lim_{z \to +0} w_z(x, y, z) = 0$ according to the condition. Hence, $w_n^\circ = 0$ in the transition region.

Eqs.(3.2) take the form

$$v_n^\circ \frac{\partial \mathbf{v}_\tau^\circ}{\partial n} = \delta \mathbf{w}_\tau^\circ - \mathbf{v}_\tau^\circ, \quad \frac{\partial p_f^\circ}{\partial n} = -v_n^\circ, \quad \frac{\partial p_s^\circ}{\partial n} = 0, \quad \frac{\partial v_n^\circ}{\partial n} = 0 \tag{3.6}$$

($\mathbf{w}_\tau^\circ = \mathbf{w}_\tau^\circ(\eta,\ \vartheta,\ n)$ is an arbitrary function).

The integration of the second and fifth equations of (3.6) has been considered above in Sect.1.

$$p_s^\circ = p_s^\circ(\eta,\ \vartheta) = p_s(\eta,\ \vartheta,\ 0) \equiv \lim_{z\to\mp 0} p_s(x,\ y,\ z)$$

follows from the fourth equation of (3.6), i.e. the pressure distribution of the solid phase in the region of the dense layer is continuous up to the boundary $n = 0$.

The jump in the pressure p_s, which is determined by the overall system of boundary conditions (1.3) is concentrated on this boundary. In the case of the known solutions (/6, 7, 10/, for example) for a fixed form of discontinuity and under the assumption that the flow fields and the pressures of the phases are stationary, the boundary condition for the pressure in the disperse phase cannot be satisfied on the whole of the surface of discontinuity but only locally in the neighbourhood of some of its points. This enables one to estimate the value of the velocity U_s, i.e. the quantity which occurs in the equations of the parameter δ.

The solution of the first equation of (3.6) is representable in the form

$$\begin{gathered}\mathbf{v}_\tau^{\circ\pm}(\eta,\vartheta,n) = \boldsymbol{\varphi}^\pm(\eta,\vartheta)\exp(-n/f(\eta,\vartheta)) + \mathbf{I}^\pm(\eta,\vartheta,n)\\ \mathbf{I}^\pm(\eta,\vartheta,n) = \\ \frac{1}{f(\eta,\vartheta)}\exp\left(-\frac{n}{f(\eta,\vartheta)}\right)\int\limits_{\substack{-\infty,\ f>0\\ +\infty,\ f<0}}^{n}\exp\left(\frac{n'}{f(\eta,\vartheta)}\right)\boldsymbol{\chi}^\pm(\eta,\vartheta,n')\,dn'\\ \boldsymbol{\chi}^\pm(\eta,\vartheta,n) \equiv \delta\mathbf{w}_\tau^\circ(\eta,\vartheta,n)\end{gathered} \tag{3.7}$$

Here and everywhere subsequently the plus sign denotes parameters on segments where the fluid phase flows in ($f > 0$), while the minus sign denotes segments where the fluid phase flows out of the dense layer ($f < 0$)

Unlike the model in Sect.1°, the distribution of $\mathbf{v}_\tau^\circ$ in (3.7) now depends on two arbitrary functions: $\boldsymbol{\varphi}^\pm(\eta,\ \vartheta)$ and $\boldsymbol{\chi}^\pm(\eta,\ \vartheta,\ n)$, which enables one to join this distribution to the solutions $\mathbf{v}_\tau$ and $\mathbf{v}_\tau'$ outside of the transition layer.

The behaviour of the function $\mathbf{I}^\pm(\eta,\vartheta,n)$ as $n\to 0,\ \infty$ is determined by certain limiting and differential properties of the distribution of the tangential component of the velocity of the solid phase in the transition layer region. Let us assume, for example, that the functions $\boldsymbol{\chi}^\pm(\eta,\vartheta,n)$ as $n\to 0$ are representable in the form of power series which are uniformly convergent with respect to η and ϑ, i.e. they are analytical close to the plane $n = 0$. In this case

$$\begin{gathered}\lim_{n\to 0}\mathbf{I}^\pm(\eta,\vartheta,n) = \sum\nolimits^\pm(\eta,\vartheta)\\ \sum\nolimits^\pm(\eta,\vartheta) = \sum_{k=0}^{\infty}(-1)^k f^k(\eta,\vartheta)\frac{\partial^k}{\partial n^k}\boldsymbol{\chi}^\pm(\eta,\vartheta,n)\big|_{n=0}\end{gathered} \tag{3.8}$$

Assuming that the functions $\boldsymbol{\chi}^\pm(\eta,\ \vartheta,\ n)$ have a limit uniformly with respect to η and ϑ as $n\to +\infty$ and are representable by convergent asymptotic series, we obtain

$$\lim_{n\to+\infty,\ f\gtrless 0}\mathbf{I}^\pm(\eta,\vartheta,n) = \mathbf{a}_0^\pm(\eta,\vartheta) \equiv \lim_{n\to+\infty,\ f\gtrless 0}\boldsymbol{\chi}^\pm(\eta,\vartheta,n) \tag{3.9}$$

We find on the basis of expressions (3.8) and (3.9) and from the matching conditions (3.3) that, in the case of the functions $\boldsymbol{\varphi}^\pm(\eta,\vartheta),\ \boldsymbol{\chi}^\pm(\eta,\vartheta,n)$, the relationships

$$\begin{gathered}f(\eta,\vartheta) > 0,\quad \mathbf{a}_0^+(\eta,\vartheta) = \mathbf{v}_\tau^+(\eta,\vartheta,0),\\ \boldsymbol{\varphi}^+(\eta,\vartheta) = \mathbf{v}_\tau'(\eta,\vartheta,0) - \sum\nolimits^+(\eta,\vartheta)\\ f(\eta,\vartheta) < 0,\quad \mathbf{a}_0^-(\eta,\vartheta) = \mathbf{v}_\tau^-(\eta,\vartheta,0)\\ \boldsymbol{\varphi}^-(\eta,\vartheta) = 0,\quad \sum\nolimits^-(\eta,\vartheta) = \mathbf{v}_\tau'(\eta,\vartheta,0)\end{gathered} \tag{3.10}$$

must be satisfied on segments into and out of which the fluid flows respectively and the series are assumed to be uniformly convergent with respect to η and ϑ.

It follows from formulae (3.10) that the need to satisfy the conditions for the asymptotic matching of the tangential components of the velocity of the fluidizing agent on the boundaries of the transition layer region when the gas flows out of the dense layer ($f < 0$) imposes more rigorous constraints on the function $\boldsymbol{\chi}^\pm(\eta,\vartheta,n)$ describing the distribution of the tangential velocity of the solid phase in the transition layer, than in the case of segments into which there is a flow of gas where it is only required that

$$\lim_{n\to+\infty}\boldsymbol{\chi}^+(\eta,\vartheta,n) = \mathbf{v}_\tau^+(\eta,\vartheta,0)$$

As an example, let us consider a simple model distribution of the tangential velocity of the disperse phase in the transition layer $\chi = \delta \mathbf{w}_\tau^\circ$ in the form

$$f(\eta, \vartheta) > 0, \quad \chi^+(\eta, \vartheta, n) = \mathbf{a}_0^+(\eta, \vartheta) + \mathbf{b}_0^+(\eta, \vartheta) e^{-n}$$
$$f(\eta, \vartheta) < 0, \quad \chi^-(\eta, \vartheta, n) = \mathbf{a}_0^-(\eta, \vartheta) + \mathbf{b}_0^-(\eta, \vartheta) e^{-n}$$

We shall have

$$\begin{aligned} &f > 0, \ \mathbf{a}_0^+(\eta, \vartheta) = \mathbf{v}_\tau^+(\eta, \vartheta, 0), \quad \varphi^+(\eta, \vartheta) = \Delta_\tau^+(\eta, \vartheta) - \\ &\quad \mathbf{b}_0^+(\eta, \vartheta)(1 - f(\eta, \vartheta))^{-1} \\ &f < 0, \ \mathbf{a}_0^-(\eta, \vartheta) = \mathbf{v}_\tau^-(\eta, \vartheta, 0), \quad \varphi^-(\eta, \vartheta) = 0, \\ &\mathbf{b}_0^-(\eta, \vartheta) = \Delta_\tau^-(\eta, \vartheta)(1 - f(\eta, \vartheta)) \end{aligned} \tag{3.11}$$

on segments into which and out of which the gas flows respectively subject to the condition $|f(\eta, \vartheta)| < 1$ uniformly with respect to η and ϑ (we recall that $f(\eta, \vartheta)$ is the dimensionless normal component of the velocity of the fluid phase in the region of the dense layer, i.e. $|f(\eta, \vartheta)| \sim 1$). In relationships (3.11), $\Delta_\tau^\pm(\eta, \vartheta) = \mathbf{v}_\tau^\pm(\eta, \vartheta, 0) - \mathbf{v}_\tau'(\eta, \vartheta, 0)$ is the jump in the tangential component of the velocity of the gas on passing across the discontinuity.

We note that the function $\chi^\pm(\eta, \vartheta, n)$ in accordance with (3.11) is uniquely determined by the matching conditions on segments where there is an outflow of fluid while $\mathbf{b}_0^+(\eta, \vartheta)$ is, generally speaking, arbitrary on segments where there is an inflow of fluid. If it is required that the tangential component of the velocity of the solid phase should be continuous on the boundary $n = 0$ (this requirement is additional and it does not follow that it is necessary from the boundary conditions (1.3) since, when $w_n|_{z=0} = w_n'|_{z=0} = 0$, the jump in $\mathbf{w}_\tau$ on the discontinuity can be arbitrary), we obtain

$$\mathbf{b}_0^+(\eta, \vartheta) = \delta \mathbf{w}_\tau'^+(\eta, \vartheta, 0) - \mathbf{a}_0^+(\eta, \vartheta) = \delta \mathbf{w}_\tau'^+(\eta, \vartheta, 0) - \mathbf{v}_\tau^+(\eta, \vartheta, 0)$$

It is known /5/ that, in the problem of the motion of a spherical bubble in a pseudo-fluidized layer, the velocity fields outside the bubble possess the property $\mathbf{v}_\tau(\eta, \vartheta, 0) = \delta \mathbf{w}_\tau(\eta, \vartheta, 0)$, whence it follows that

$$\mathbf{v}_\tau(\eta, \vartheta, 0) = \mathbf{a}_0(\eta, \vartheta) \equiv \lim_{n \to +\infty} \delta \mathbf{w}_\tau^\circ(\eta, \vartheta, n) = \lim_{z \to +0} \delta \mathbf{w}_\tau(x, y, z)$$

i.e. the continuity of the tangential velocity of the solid phase on the boundary $n = +\infty$ of the transition region with the dense layer (which is obviously valid both for segments into which there is an inflow as well as for segments out of which a flow occurs).

An important difference between the models considered in Sects.1° and 2° is that, in the case of an immobile disperse phase, the flow behind the rear edge of the inhomogeneity $(f < 0)$ propagates without the formation of a transition layer /4/. As was noted above, the constraints of the model do not enable one to continuously match the distributions $\mathbf{v}_\tau(x, y, z)$, $\mathbf{v}_\tau^\circ(\eta, \vartheta, n)$, $\mathbf{v}_\tau'(x, y, z)$ on the boundaries of the transition layer $(f > 0)$.

In the model with a mobile disperse phase, a transition layer is formed on the surface of discontinuity regardless of the direction in which the discontinuity intersects the flow of the fluid phase. In the zeroth approximation with respect to the thickness of the transition layer, the condition of continuity of the tangential component of the velocity of the fluid phase upon intersecting the discontinuity is not necessary along the whole of its surface.

3°. *Discontinuity in the pseudofluidized layer with a continuous concentration distribution of the disperse phase in the transition region.* In the model being considered, the system of transition layer equations has the form

$$\begin{aligned} &v_n^\circ \frac{\partial \mathbf{v}_\tau^\circ}{\partial n} = \sigma(\eta, \vartheta, n)(\delta \mathbf{w}_\tau^\circ - \mathbf{v}_\tau^\circ), \quad v_n^\circ \frac{\partial v_n^\circ}{\partial n} = -\frac{\partial p_f^\circ}{\partial n} - \sigma(\eta, \vartheta, n) v_n^\circ \\ &\frac{\partial p_s^\circ}{\partial n} = 0, \quad \frac{\partial}{\partial n}(\varepsilon^\circ v_n^\circ) = 0, \quad w_n^\circ = 0 \\ &\rho^\circ = \rho^\circ(\rho, \rho') = \begin{cases} \rho, & n \to +\infty \\ \rho', & n \to 0 \end{cases} \end{aligned} \tag{3.12}$$

where $\delta \mathbf{w}_\tau^0(\eta, \vartheta, n) \equiv \chi(\eta, \vartheta, n)$ is an arbitrary function (as previously, we consider discontinuities which are impermeable to the particles).

The introduction of a continuous concentration distribution in the transition layer which, in the integration of system (3.12), provides additional arbitrariness in the form of a certain function ρ° with the above-mentioned properties as $n \to 0, +\infty$, enables one to obtain a pressure distribution and a distribution of the normal component of the velocity of the fluidizing agent which are continuous on the boundaries of the transition region. In particular, in the latter case $v_n^\circ = f(\eta, \vartheta)/\varepsilon^\circ$, where $f(\eta, \vartheta) = \varepsilon v_n(\eta, \vartheta, 0) = \varepsilon' v_n'(\eta, \vartheta, 0)$ in accordance with the matching conditions (3.3) and the first condition (1.3). It is assumed that the function $\rho^\circ(\eta, \vartheta, n)$ and the functions ε°, $\sigma(\rho^\circ, \rho)$ associated with it satisfy the additional constraints imposed on them during the construction of the distribution of v_z and p_f which are continuous "when account is taken" of the transition layer.

In the case of the velocity component of the gas which is tangential to the discontinuity, we obtain

$$\mathbf{v}_\tau^{\circ\pm}(\eta,\vartheta,n)=\boldsymbol{\varphi}^\pm(\eta,\vartheta)E(\eta,\vartheta,n)+\mathbf{I}^\pm(\eta,\vartheta,n)$$

$$\mathbf{I}^\pm(\eta,\vartheta,n)=\frac{E(\eta,\vartheta,n)}{f(\eta,\vartheta)}\int\limits_{\substack{-\infty,\,f>0\\+\infty,\,f<0}}^{n}E^{-1}(\eta,\vartheta,n')\boldsymbol{\varkappa}^\pm(\eta,\vartheta,n')\,dn'$$

$$E(\eta,\vartheta,n)=\exp\Big[-\frac{1}{f(\eta,\vartheta)}\int\limits_0^n\sigma(\eta,\vartheta,n_1)\,\varepsilon^\circ(\eta,\vartheta,n_1)\,dn_1\Big]$$

$$\boldsymbol{\varkappa}^\pm(\eta,\vartheta,n)=\boldsymbol{\chi}^\pm(\eta,\vartheta,n)\,\sigma(\eta,\vartheta,n)\,\varepsilon^\circ(\eta,\vartheta,n)$$

which becomes relationship (3.7) when $\sigma(\rho^\circ,\rho)=1$.

Let us assume that the limits

$$\mathbf{I}_\infty^\pm(\eta,\vartheta)=\lim_{n\to+\infty}\mathbf{I}^\pm(\eta,\vartheta,n),\quad \mathbf{I}_0^\pm(\eta,\vartheta)=\lim_{n\to0}\mathbf{I}^\pm(\eta,\vartheta,n) \tag{3.13}$$

exist.

By virtue of the arbitrariness of the function $\boldsymbol{\chi}^\pm(\eta,\vartheta,n)=\delta\mathbf{w}_\tau^\circ(\eta,\vartheta,n)$ for impermeable discontinuities, the requirements of (3.13) are exceedingly general which enables one to construct a distribution of $\mathbf{v}_\tau$ which is continuous over the whole of the flow region if the solution outside of the transition layer is known. At the same time, it follows from the matching conditions (3.3) that

$$f(\eta,\vartheta)>0,\quad \mathbf{v}_\tau^+(\eta,\vartheta,0)=\mathbf{I}_\infty^+(\eta,\vartheta),$$
$$\boldsymbol{\varphi}^+(\eta,\vartheta)=\mathbf{v}_\tau'(\eta,\vartheta,0)-\mathbf{I}_0^+(\eta,\vartheta)$$
$$f(\eta,\vartheta)<0,\quad \mathbf{v}_\tau^-(\eta,\vartheta,0)=\mathbf{I}_\infty^-(\eta,\vartheta),\quad \boldsymbol{\varphi}^-(\eta,\vartheta)=0,$$
$$\mathbf{v}_\tau'(\eta,\vartheta,0)=\mathbf{I}_0^-(\eta,\vartheta)$$

Let us estimate the dimensionless thickness Δ of the transition layer on the discontinuity. It follows from relationships (2.1) that $\Delta\sim\mathrm{Ar}^*\sim\lambda^{1-m}$. It is obvious from this that it is possible to choose the quantities $m\in(0,1)$ and λ in order that the thickness of the transition layer will be sufficiently large $(\Delta/\varkappa\sim\lambda^{-m})$ with respect to the distance between the phases, $\Delta/\varkappa$, $\varkappa\sim\lambda$, so that the condition for the applicability of the continuum approach to the description of the flow in the transition layer will be complied with and, at the same time, sufficiently small compared with the size of the macroinhomogeneity (packet, bubble). The latter fact enables one to neglect the presence of a transition region in describing the flow from the discontinuity.

In the zeroth approximation with respect to the thickness of the transition layer, the system of boundary conditions (1.3) must be changed. In particular, on a "strong" discontinuity which is impermeable to the disperse phase $(\rho\gg\rho')$, the tangential component of the velocity of the fluidizing agent may experience an arbitrary jump even in the case of a non-zero flux of gas through the surface of discontinuity $(v_n\neq0)$. Then change in the parameters p_f, v_n, $\mathbf{v}_\tau$, $P_f=p_f+{}^1/_2\,v_n^2$ on intersecting the discontinuity is shown schematically in Fig.1 for two cases: taking account of the transition layer and under the assumption that there is an infinitely thin transition zone.

The modified system of boundary conditions on a discontinuity of the type being considered in the approximation $\Delta=0$ has the form

$$[\varepsilon v_z]=0,\quad w_z=0,\quad w_z'=0,\quad [p_f]=-{}^1/_2d_f[v_z^2] \tag{3.14}$$
$$[p_f+p_s+d_f\varepsilon v_z^2]=0$$

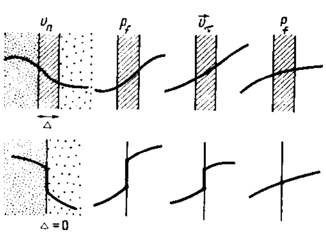


Fig.1

In the model of a small but finite inertia, it follows that the change in the dynamic pressure of the fluid phase on traversing the discontinuity should be taken into account in the construction of the flow field of the gas in the region where there is a low concentration of particles (see paragraph 2, (2.2)).

In the model of an inertialess fluidizing agent $(d_f=0)$ it is approximately assumed /5, 8/ that

$$[\varepsilon v_z]=0,\quad w_z=0,\quad p_f=\mathrm{const},$$
$$p_\Sigma=\mathrm{const},\quad p_\Sigma=p_f+p_s$$

on the boundary of the layer with the region filled with the pure gas.

Within the framework of this model, information concerning the flow pattern of the fluid phase outside the dense layer is, generally speaking, lost.

4. Motion of a bubble in a pseudofluidized layer. On the basis of the results which have been obtained we shall consider the model problem of the quasistationary motion, in a pseudofluidized layer, which is constant with respect to the shape and dimensions of the spherical cavity which is free from solid particles (/6-8/,/12/, et al.). The physical formulation of the problem of finding the flow fields of the fluidizing agent and the disperse phase in the neighbourhood of a bubble involves the initial equations (1.1) $(\partial/\partial t = 0)$ with the assumption that the interaction between the phases is linear (see (1.2), (1.5)), the boundary conditions (3.14) which must be satisfied on the spherical surface of the bubble which is a surface of discontinuity in the concentration of the particles of the type considered above, and also the conditions for the uniformity of the gas and particle flows at large distances for the inhomogeneity.

Use is made of a spherical coordinate system (r, θ, φ) associated with the centre of the bubble, the polar axis of which is parallel to the external mass force vector.

Within the framework of the assumed model the problem of the motion of the phases can be solved without the use of a boundary condition for the pressure of the disperse phase on the surface of the bubble. We note that representations concerning the physical nature of the pressure of a pseudogas of particles and also concerning phenomena which are analogous to surface tension in liquids cannot be considered as having been formulated in the case of pseudofluidized system at the present time. The correct formulation of the condition for the balance of the total normal stresses in a disperse system (which has been written in (3.14) in a simplified form) involves serious difficulties and requires additional investigation.

By applying the curl operator to the initial equations we arrive at the following relationship for the dimensionless flow functions of the fluid and solid phases $(\bar{\psi} = \psi/v_0 a^2$, a is the radius of the bubble, $\bar{r} = r/a$, the dashes are subsequently omitted) which, outside the bubble, satisfy the condition for the homogeneity of the flows of the phases at infinity (V_b is the velocity of levitation of the bubble)

$$r > 1, \quad \psi_f = (Mr^{-1} + (\delta - 1) r^2/2) \sin^2\theta, \quad \psi_s = (Gr^{-1} + \delta r^2/2) \sin^2\theta \tag{4.1}$$
$$r < 1, \quad \psi_f' = (A + Cr^2) r^2 \sin^2\theta, \quad \delta = V_b/v_0$$

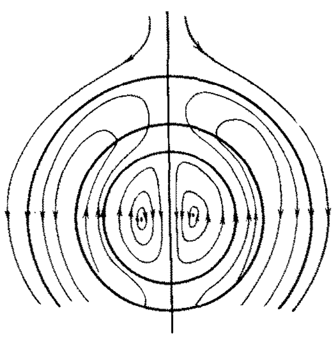
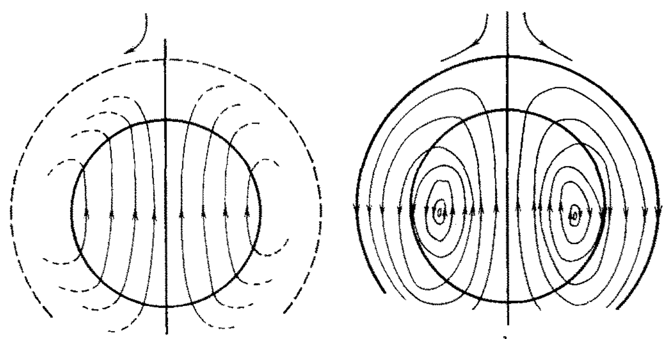


Fig.2



a b

Fig.3

With the help of the boundary conditions (3.14), we obtain the following algebraic system for determining the unknown coefficients M, G, A and C:

$$\varepsilon (2M + \delta - 1) = 2 (A + C), \quad G + \delta/2 = 0 \tag{4.2}$$
$$M - G + 1 = 0, \quad (M - \delta + 1)^2 = 4 (A^2 - AC - C^2)$$

and, by solving these, we find

$$G = -\delta/2, \quad M = -(\delta/2 + 1) \tag{4.3}$$
$$C_{1,2} = -\frac{9\varepsilon}{4}(1 \pm \sqrt{b}), \quad A_{1,2} = \frac{3\varepsilon}{4}(1 \pm 3\sqrt{b}), \quad b = \frac{\delta^2 + 5\varepsilon^2}{9\varepsilon^2}$$

The two different sets of coefficients A and C in the solution correspond to the two real roots of the last equation in (4.3) which has the form

$$(9b - 5)(X + 1)^2 = 4(X^2 - X - 1), \quad X = A/C$$

The plus sign corresponds to the first root $(X = X_2, |X_2| > 1)$ and the minus sign in the expressions for the coefficients $A_{1,2}$ and $C_{1,2}$ corresponds to the second root $(X = X_1, |X_1| < 1)$.

We emphasize that, in accordance with formulae (4.1) and (4.2), the flow pattern of the phase outside the bubble is identical to that found in the Davidson model /6/. The only difference between the results which are discussed below and those obtained in /6/ concerns the nature of the flow of the fluidizing agent in the region $r < 1$ within the bubble.

We now consider the possibility of forming closed stream lines of the fluid phase within the bubble. It follows from the third equality of (4.1) that, in order for this to happen, the condition $A_{1,2} + C_{1,2}r^2 = 0$ must be satisfied. When allowance is made for relationships (4.3), the latter equation leads to the following expression for the dimensionless radius of the spherical boundary of the cloud within the bubble:

$$\bar{a}_c'^2 = \left(\frac{a_c'}{a}\right)^2 = -\frac{A_{1,2}}{C_{1,2}} = 1 - \frac{2}{3(1 \pm \sqrt{b})}$$

It is obvious that a cloud with circulating gas can only exist in the region being considered in the case when $\bar{a}_c' = \bar{a}_{c1}', |\bar{a}_{c1}'| < 1$. The second solution yields the dependence $\bar{a}_c' = \bar{a}_{c2}', |\bar{a}_{c2}'| > 1$ which physically means that there are no closed flows of the fluidizing agent which form a spherical cloud within the packet. We note that the possibility of a spherical cloud of closed circulation, similar to a Hill vortex, arising within bubbles is indirectly confirmed, for example, by experimental data from a study of the mass exchange of bubbles in a pseudofluidized layer /13/ (also see /6/). As the magnitude of the parameter b increases, the cloud rapidly increases in size and occupies practically the whole of the interior of the bubble.

The region of closed circulation of the gas in a bubble when $b \cong 2.33$ $(\varepsilon = 0.5, \delta = 2, \bar{a}_c' = 0.858)$ is shown schematically in Fig.2. At the same time, the cloud $(\bar{a}_c = ((\delta + 2)/(\delta - 1))^{1/2} \cong 1.58)$ also exists outside the bubble.

The change in the pattern of the flow lines within the bubble which corresponds to the second solution of (4.3), is schematically represented in Fig.3, a, b. In this case, the interior of the bubble is circulating when $5/9 < b < 25/9$. In the case of rapidly rising bubbles ($b > 25/9$, Fig.3, b) conditions are realized for the creation of a toroidal-shaped annular vortex within the cavity which covers the bubble along its equator. The relationship $d_m = 1 - (2/3)^{1/2}(\sqrt{b} - 1)^{-1/2}$ holds for the mean diameter of the vortex. The size of the vortex like the size of the spherical cloud, has a tendency to increase as the velocity of levitation of the bubble increases.

Hence, the model employed in this paper for the flow field of a fluid within a bubble allows two solutions of the initial system of Eqs.(1.1) with boundary conditions (3.14) which correspond to two different patterns of flow lines of the fluid phase when $r < 1$. The first solution corresponds to the occurrence within the bubble of a spherical cloud of the fluid phase, the dimensions of which are determined by the size of the bubble and the properties of the disperse system. The second solution is characterized by the circulation of the slowly levitating bubbles and by the formation within them of a pre-equatorial annular vortex in the case when the levitation velocity is fairly large. This solution is similar to a known extent to the solution in /14/ which was obtained, however, within the framework of the Davidson model without taking account of the jump in the pressure of the fluidizing agent on the boundary of the bubble and with the improper use of the condition for the continuity of the tangential component of the velocity of the gas on this boundary.

The analysis presented above was carried out without the assumption made by Davidson concerning the constancy of the gas pressure within the bubble, and takes into account the inertia of the fluid phase and the jump in its dynamic pressure on crossing the surface of the bubble. Within the framework of the adopted model, the question as to which of the two solutions which have been found is preferable remains open. In reality, both of the above-mentioned possibilities can obviously be realized. Verification of the adequacy of the results which have been obtained requires more refined experiments with the aim of studying the flow pattern of the fluidizing agent within the bubbles in pseudofluidized systems.

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CALCULATION OF THE FORCE AND MOMENT OF FORCES ACTING ON A DROP IN AN ARBITRARY NON-STEADY FLOW OF A VISCOUS FLUID*

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The Oseen point force method /1-3/ which differs from the methods used earlier in similar problems, is used to obtain formulas for the force and moment of forces acting on a spherical drop in an inhomogeneous non-steady flow of viscous incompressible fluid. In special cases the results can be reduced to well-known results.

Earlier, the non-steady motion of a rigid particle in an inhomogeneous non-steady flow was considered in /4, 5/, its rotation in /6, 7/, the conditions of slippage at the surface in /5, 7/, and the effect of the specified external forces in /8, 9/. The corresponding stationary problem was studied in /10, 11/ and the non-steady motion of a drop in uniform non-steady flow in /12–14/.

1. Formulation of the problem. A liquid sphere of viscosity μ', density ρ' and constant radius a, moves with velocity $\mathbf{u}(t)$ through an incompressible medium of viscosity μ and density ρ. The problem is studied in the Stokes approximation, i.e. we consider the following linear, non-steady equations of motion of the fluid outside and inside the drop:

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